Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = 4\sin(x)\cos(x)$

$$f'(x) = 3(\cos x \cdot \cos x + \sin x(-\sin x))$$

b. $f(x) = \frac{\sqrt{3}}{4} + \frac{\sqrt{x}}{5} - \frac{5}{\sqrt{x}}$

$$f'(x) = \frac{1}{5} \cdot \frac{1}{2\sqrt{x}} - 5 \cdot \frac{-1}{2\sqrt{x^3}}$$

c. $f(x) = \frac{\ln(x)}{\tan(x)}$

$$f'(x) = \frac{\frac{1}{x} \cdot \tan x - \ln x \cdot \sec^2 x}{\tan^2 x}$$

d. $y = 3\csc(e^x)$

$$\frac{dy}{dx} = -3\csc(e^x)\cot(e^x)\cdot e^x$$

e. $y = 5^x - \log_5(x)$

$$\frac{dy}{dx} = 5^x \ln 5 - \frac{1}{x\ln 5}$$

f. $f(x) = \left(x^4 + \frac{1}{x} + e^5\right)^3$

$$f'(x) = 3\left(x^4 + \frac{1}{x} + e^5\right)^2 \cdot \left(4x^3 - x^{-2}\right)$$

g. $y = \left(x^{0.2} + \sec(x)\right)^{-2/3}$

$$\frac{dy}{dx} = -\frac{2}{3}\left(x^{0.2} + \sec x\right)^{-5/3} \cdot \left(0.2x^{-0.8} + \sec x \tan x\right)$$

h. $f(x) = \dfrac{\cos(\pi/x)}{x^2}$

$$f'(x) = \frac{-\sin\left(\frac{\pi}{x}\right) \cdot \left(-\pi x^{-2}\right) \cdot x^2 - \cos\left(\frac{\pi}{x}\right) \cdot 2x}{\left(x^2\right)^2}$$

i. $f(x) = 3\sin^{-1}(3x^3)$

$$f'(x) = \frac{3}{\sqrt{1-(3x^3)^2}} \cdot 9x^2$$

j. $f(x) = \ln\left(\frac{x^2e^x}{14x}\right)$ $= 2\ln x + x - \ln 14 - \ln x$

$f'(x) = \frac{2}{x} + 1 - \frac{1}{x}$

k. $f(x) = \frac{\sin(6)}{\sqrt[3]{\sin(x)}}$ $= \sin(6)\cdot(\sin x)^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}\sin(6)\cdot(\sin x)^{-4/3}(\cos x)$

l. Find $\frac{dy}{dx}$ for the equation $e^x + e^y = 2\sin(xy)$. You must solve for $\frac{dy}{dx}$.

$\frac{d}{dx}\left[e^x + e^y\right] = \frac{d}{dx}\left[2\sin(xy)\right] \Rightarrow e^x + e^y\cdot\frac{dy}{dx} = 2\cos(xy)\left(y + x\frac{dy}{dx}\right)$

$\Rightarrow e^x - 2\cos(xy)y = -e^y\cdot\frac{dy}{dx} + 2\cos(xy)x\frac{dy}{dx}$

$\Rightarrow \frac{e^x - 2\cos(xy)y}{-e^y + 2\cos(xy)x} = \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{e^x - 2\cos(xy)y}{-e^y + 2\cos(xy)x}}$